**Functions**

Table of Contents

[Number Line 3](#_Toc65937914)

[Functions 4](#_Toc65937915)

[Checking Function from a Graph (Vertical Line Test) 6](#_Toc65937916)

[Library of Functions 8](#_Toc65937917)

[Piecewise Defined Functions 11](#_Toc65937918)

[Even and Odd Functions 13](#_Toc65937919)

[Combining Functions: Shifting and Scaling Graphs 16](#_Toc65937920)

[Composite Functions 17](#_Toc65937921)

[Scaling the Graph of a Function 18](#_Toc65937922)

[Techniques of Graphs of Functions 18](#_Toc65937923)

[Inverse Functions 19](#_Toc65937924)

[Polynomial Functions 20](#_Toc65937925)

[Power Functions 21](#_Toc65937926)

[Rational Functions 25](#_Toc65937927)

[Vertical Asymptotes 25](#_Toc65937928)

[Horizontal Asymptote 26](#_Toc65937929)

[Oblique Asymptote 26](#_Toc65937930)

[Transcendental Functions 28](#_Toc65937931)

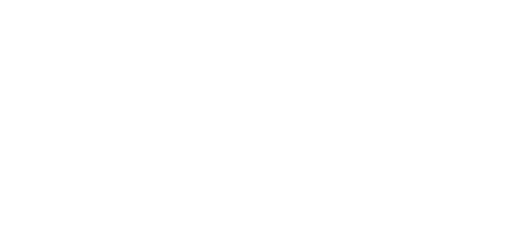
[Exponential Functions 28](#_Toc65937932)

[Logarithm Function 30](#_Toc65937933)

[Trigonometric Functions 33](#_Toc65937934)

[Graphs of Sinusoidal Functions 34](#_Toc65937935)

## Number Line



Interval – Set of numbers between and

– open interval (excluding and )

– closed interval (including and )

|  |  |  |
| --- | --- | --- |
| Interval | Set Description | Graphical Representation |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

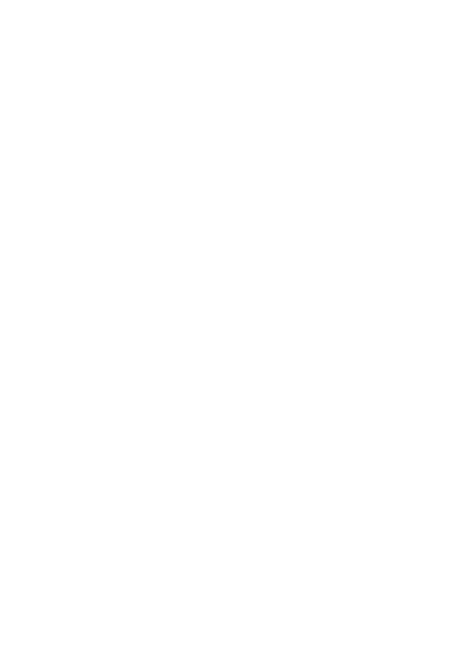
E.g.:



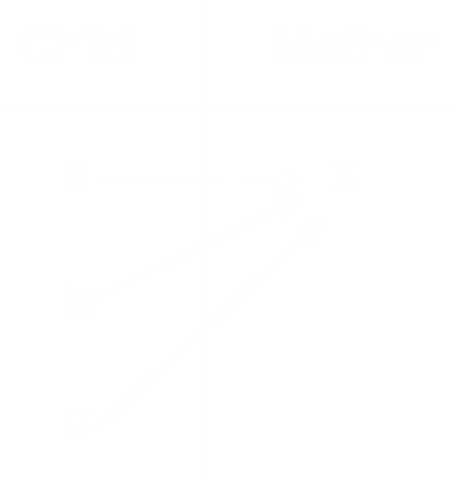
The infinite side will always have an open bracket.

## Functions

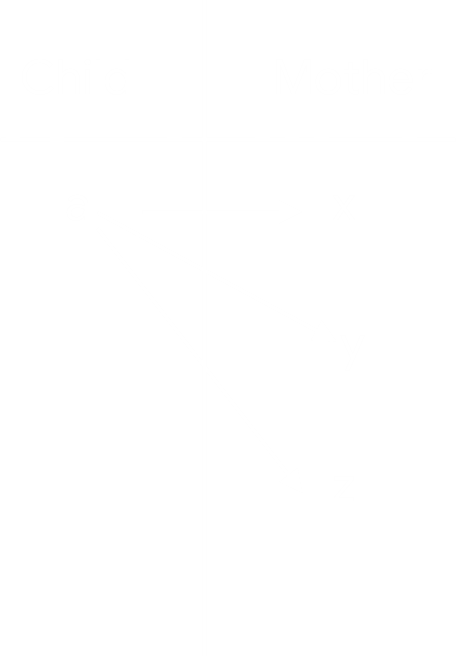
When an element of set has been associated with exactly one element of set , it is a function. Function is a special type of relation.



This is not a function, since there is one child without a mother. This is not possible in functions.



This is a function, since there can be 3 children of one mother.



This is not a function, since one child cannot have mothers.

If we imagine the child as a domain and the mother as a range, this is how the function works and we can recognize if the function is real or not.

Function: If a variable depends on a variable in such a way that each value of determines exactly one value of , then we can say that is a function of .

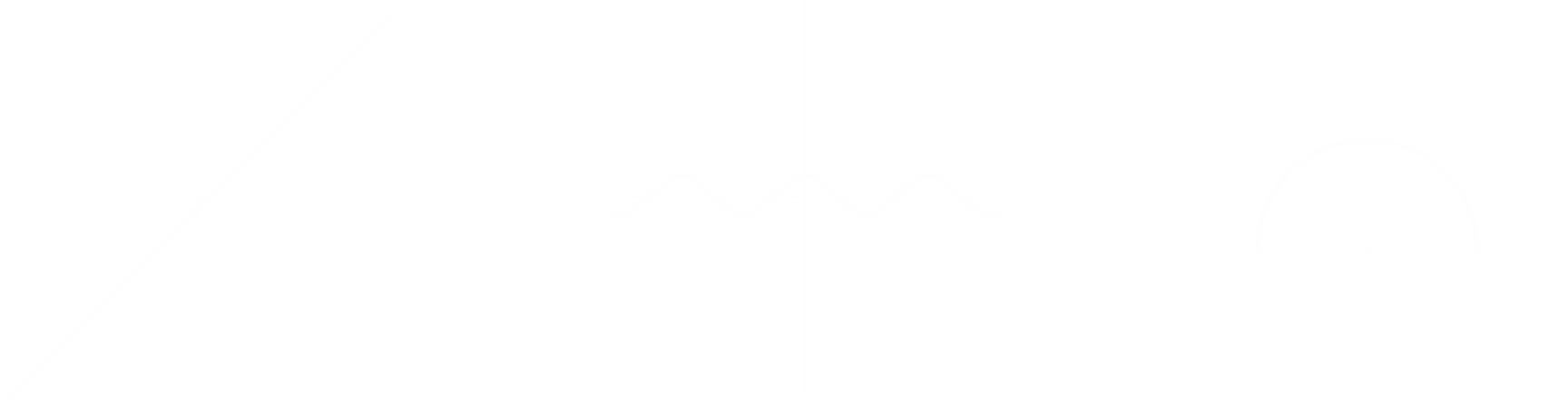
– input/independent variable, – output/dependant variable

is a function

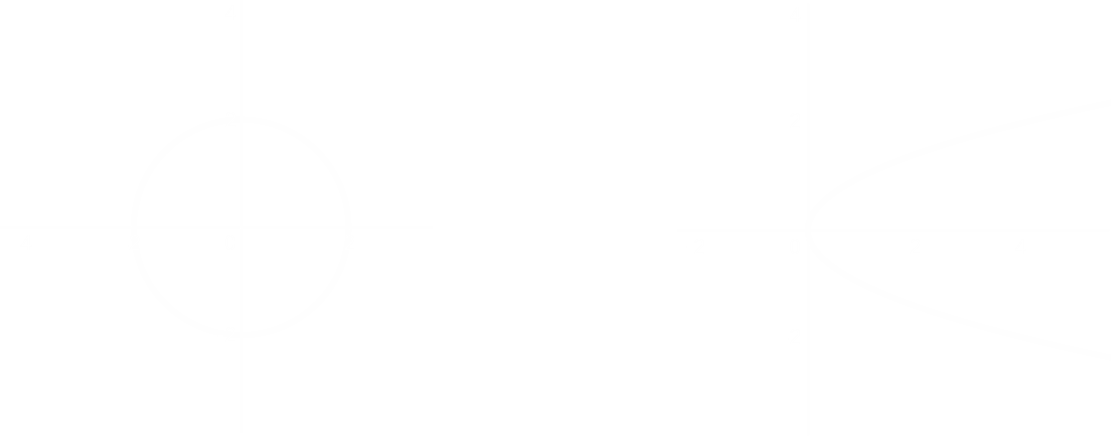
is not a function.

## Checking Function from a Graph (Vertical Line Test)

If a vertical line intersects the graph at only one point, then this graph is the graph of a function.



Graphs of functions.



Graphs of non-functions.

Domain: If any input makes the output imaginary, or undefined, then this input will not be a domain. All other real numbers can be the domain of a function.

Input – Real Output – Imaginary, Undefined ( can be any number)

Imaginary:

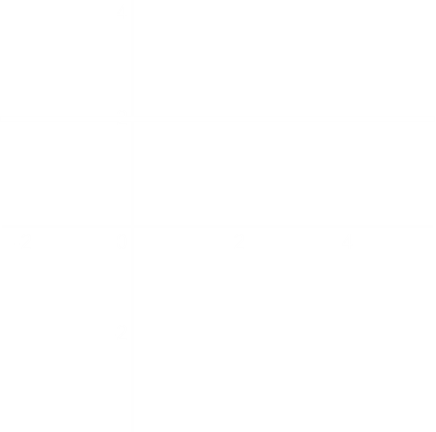
Undefined:

Find the domains from the following functions:

2. or
3. or
4. or

## Library of Functions

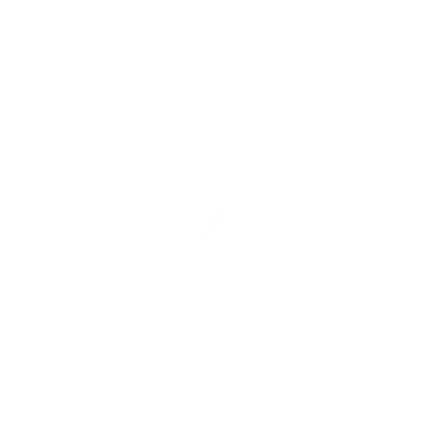
1. Constant Function



Domain:

Range:

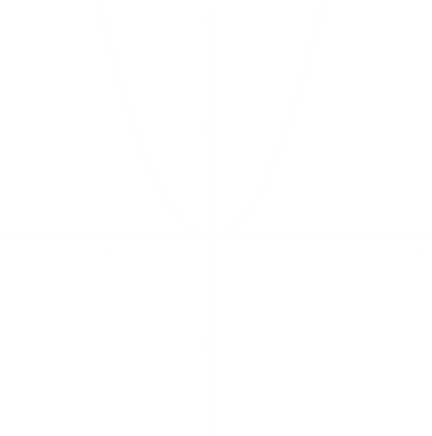
1. Identity Function



Domain:

Range:

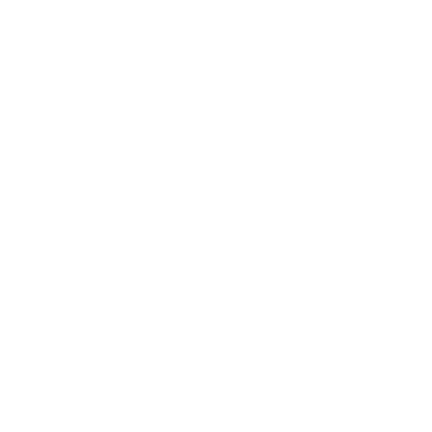
1. Square Function (also called a vertical parabola)



Domain:

Range:

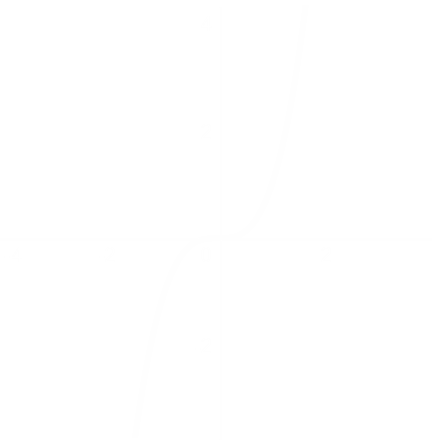
1. Square Root Function



Domain:

Range:

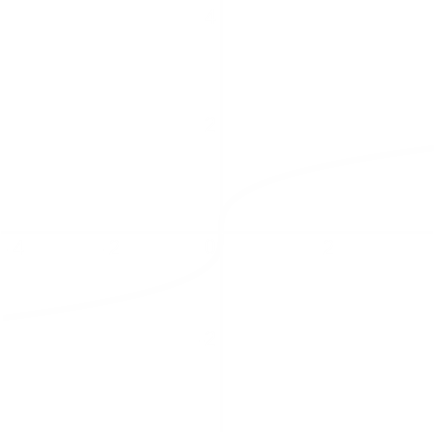
1. Cube Function



Domain:

Range:

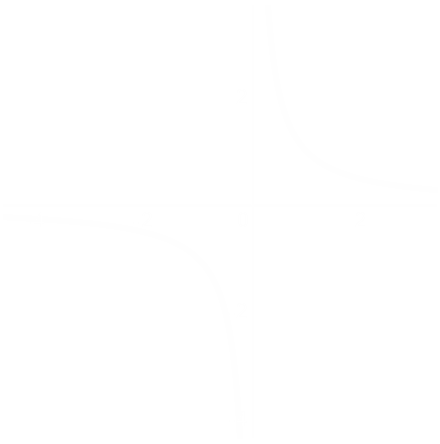
1. Cube Root Function



Domain:

Range:

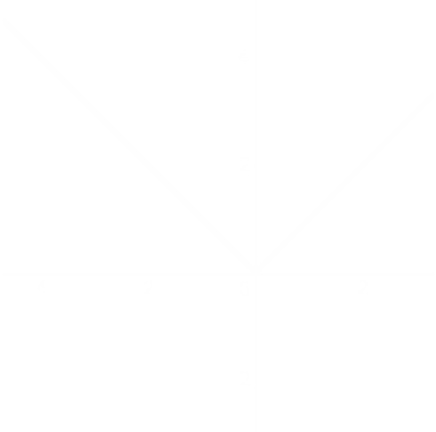
1. Reciprocal Function



Domain:

Range:

1. Absolute Value Function



Domain:

Range:

## Piecewise Defined Functions

Sometimes, a function is defined using different equations for different parts of its domain. Such a function is called a piecewise defined function.

can be written as

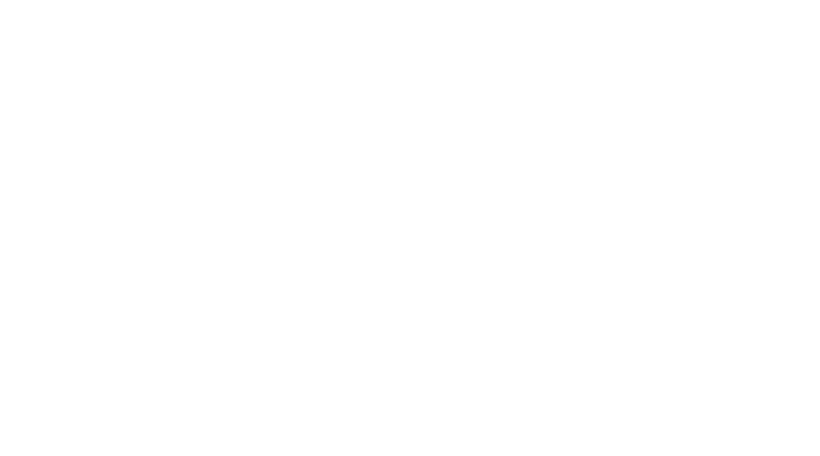
for and

for

for

for

for



Domain Range

for

for

for

for

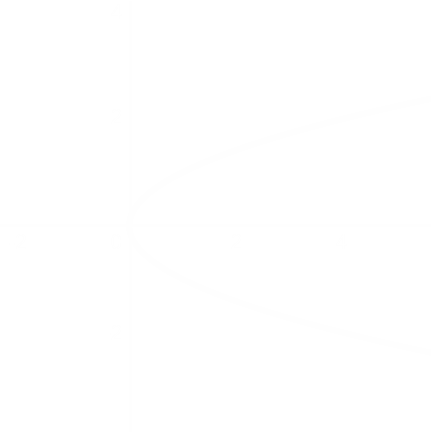
for , , i.e. (common)

for , , i.e.

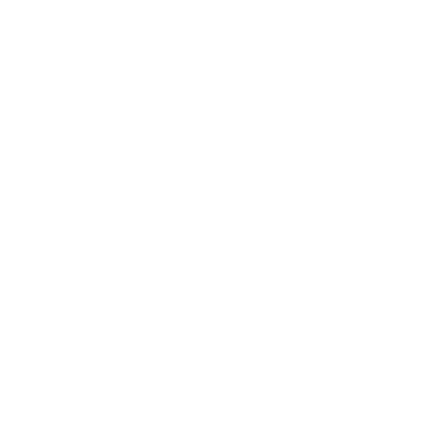
for , , i.e.

for , , i.e. no common interval

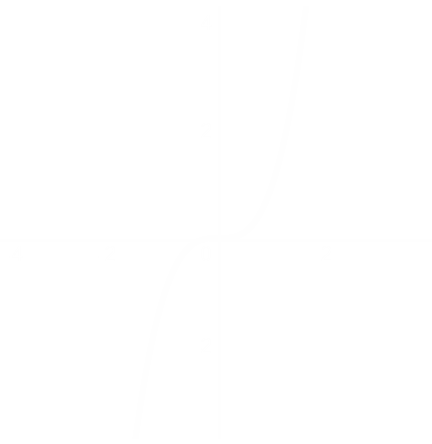
## Even and Odd Functions



Symmetrical about -axis Not a function



Symmetrical about -axis Even function

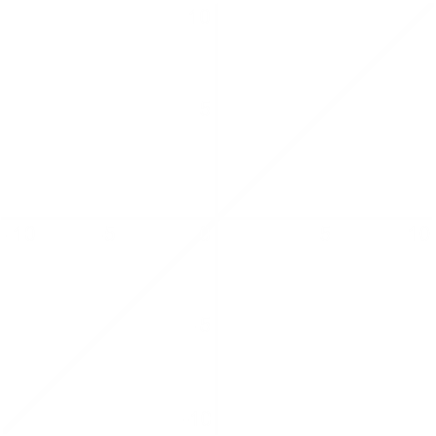


Symmetrical about origin Odd function

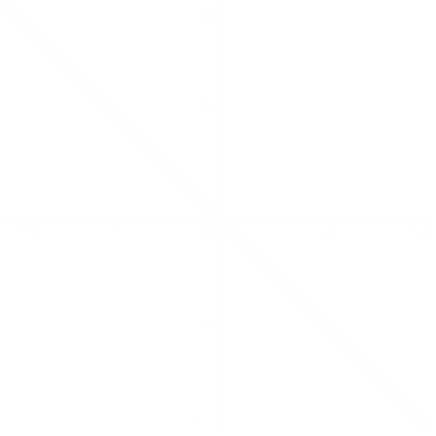
Even Functions:

Odd Functions:

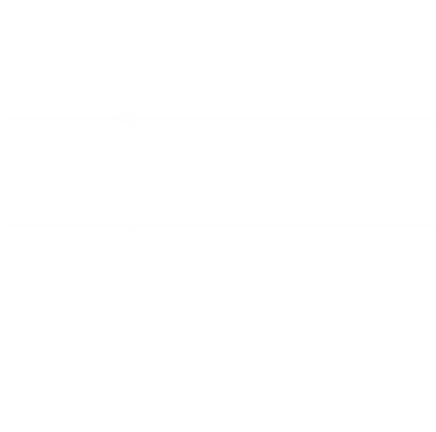
Increasing Function: input increases, output increases



Decreasing Function: input increases, output decreases



Constant Functions: input increases, output constant



## Combining Functions: Shifting and Scaling Graphs

The domains of the resulting functions is the intersection of the domains of the component functions.

## Composite Functions

composed with :

Here, is the inner function and is the outer function.

## Scaling the Graph of a Function

### Techniques of Graphs of Functions

1. Shifting

Vertical is shifted units up or down

Horizontal is shifted units right or left

1. Stretching and Compression

Vertical stretched if and compressed if

co-ordinate multiplied by

Horizontal stretched if and compressed if

co-ordinate multiplied by

1. Reflection

Vertical

Horizontal

## Inverse Functions

* Must be 1-to-1 function.
* Always reflected about .

## Polynomial Functions

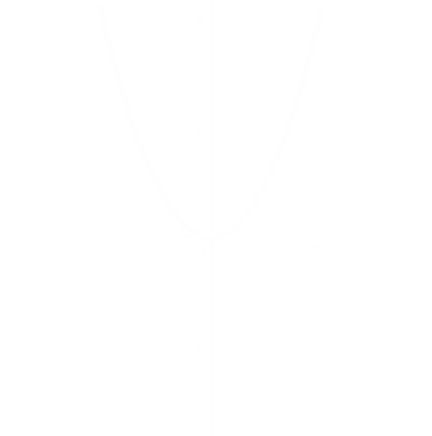
General Form:

* is a positive integer
* , , … , are real numbers
* Highest power of is called its degree
* Domain

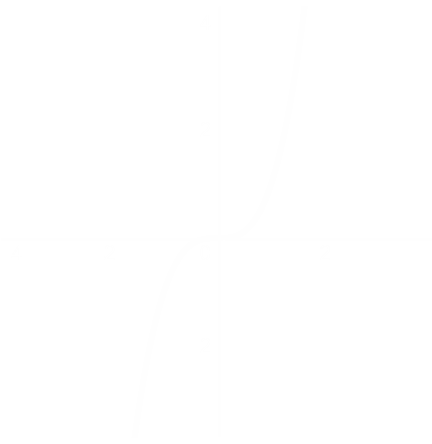
Everything is a polynomial function, even constant, linear and quadratic functions. However, if a variable has a non-positive integer power, it is not a polynomial function.

## Power Functions

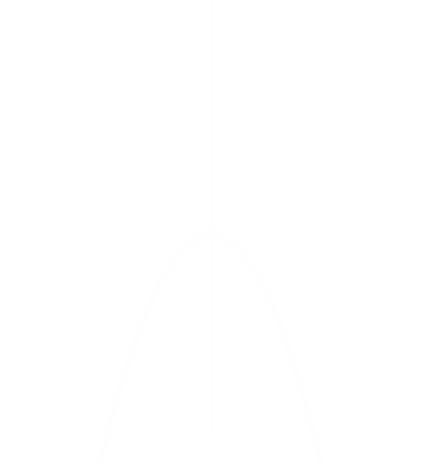
is even



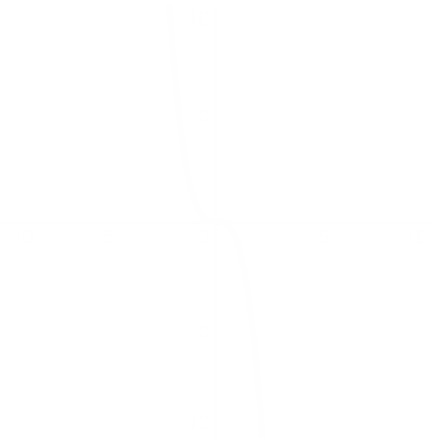
is odd



is even



is odd



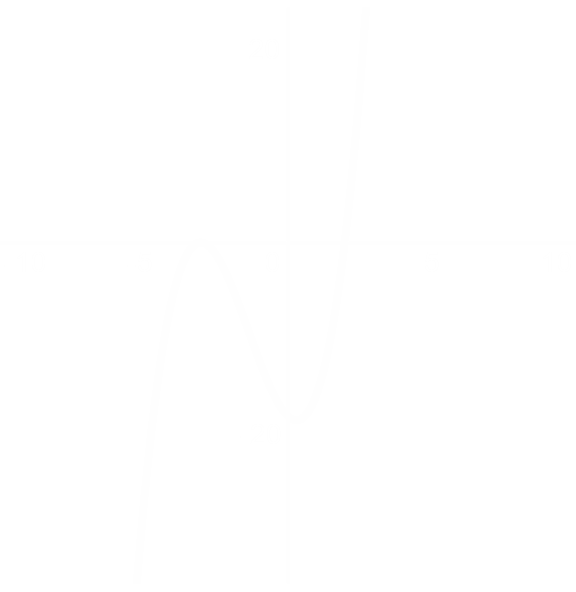
Graph the function:

The leading term dictates the end behaviour of the graph.

- The multiplicity of this term is odd, so the graph crosses the -axis at .

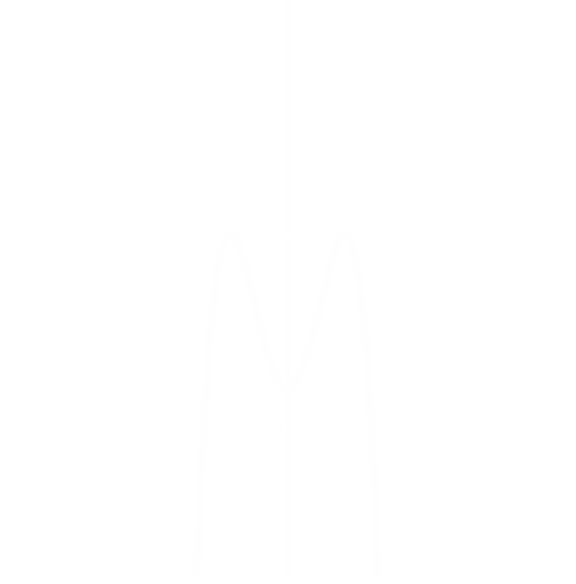
- The multiplicity of this term is even, so the graph touches the -axis at .

Thus, the graph formed is this:



Note: An -th degree function has turning points.

Interpret function from graph



The graph touches the -axis at and . So, the function should be:

However, the end behaviour of the graph shows that it should be a negative curve, so a minus sign is added to the front of the function. Thus,

## Rational Functions

Rational functions are represented by the ratio between two polynomial functions.

is a rational function.

is not a rational function since the numerator is not a polynomial function.

The speciality of rational functions is that they can have asymptotes.

There are 3 types of asymptotes:

1. Vertical asymptotes ()
2. Horizontal asymptotes ()
3. Oblique asymptotes )

### Vertical Asymptotes

If , then .

is a vertical asymptote

is a vertical asymptote.

### Horizontal Asymptote

If , then .

is a horizontal asymptote.

Rules:

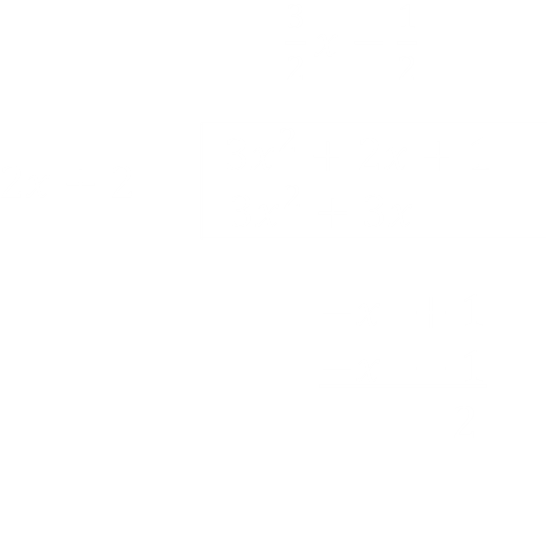
* If the degree of the numerator is less than the degree of the denominator, is the horizontal asymptote.
* If the degree of the numerator is equal to the degree of the denominator, is the horizontal asymptote. (leading coefficient is the coefficient of the term with the highest power)
* If the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote. In this scenario, an oblique asymptote may exist. An oblique asymptote and a horizontal asymptote never coexist.

### Oblique Asymptote

is the vertical asymptote.

There is no horizontal asymptote.

The oblique asymptote can be found by long division.

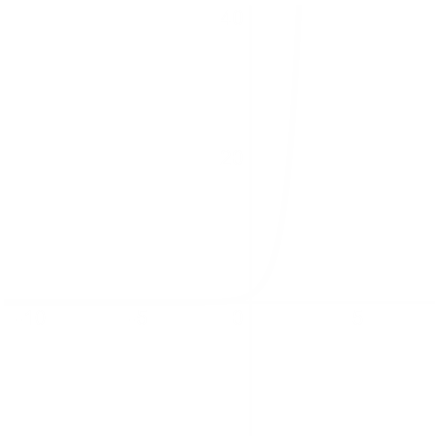


is the oblique asymptote.

## Transcendental Functions

### Exponential Functions

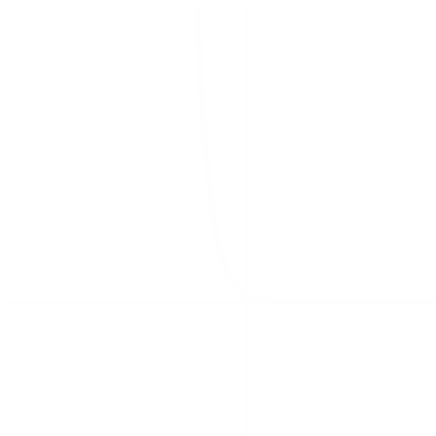
General Form: , , , is the growth factor



Domain:

Range:

Horizontal Asymptote:

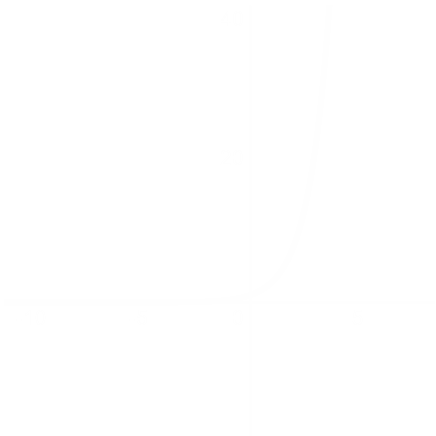


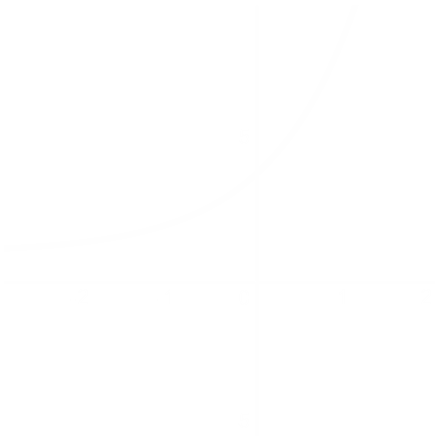
Domain:

Range:

Horizontal Asymptote:

Natural Growth Factor,





### Logarithm Function

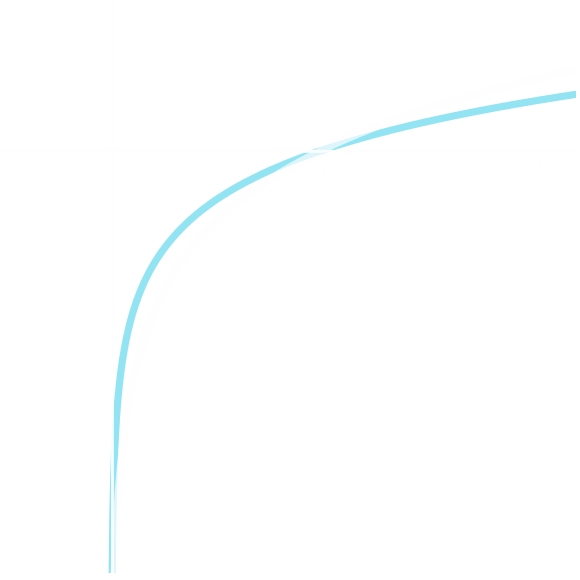
Logarithm functions are the inverse of exponential functions.

is a one-to-one function.

only if

base

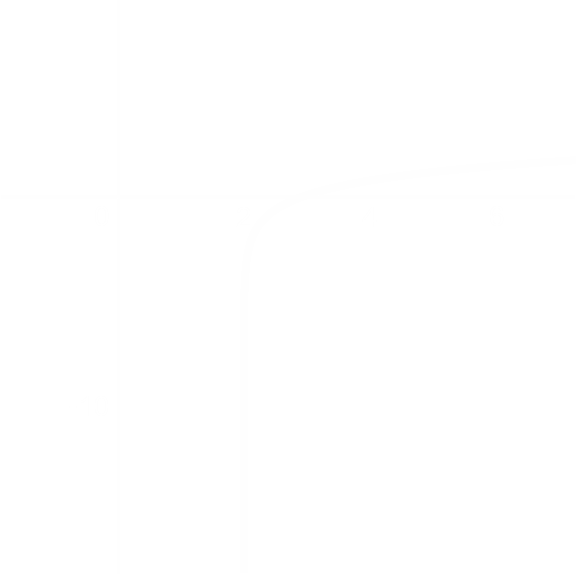
base



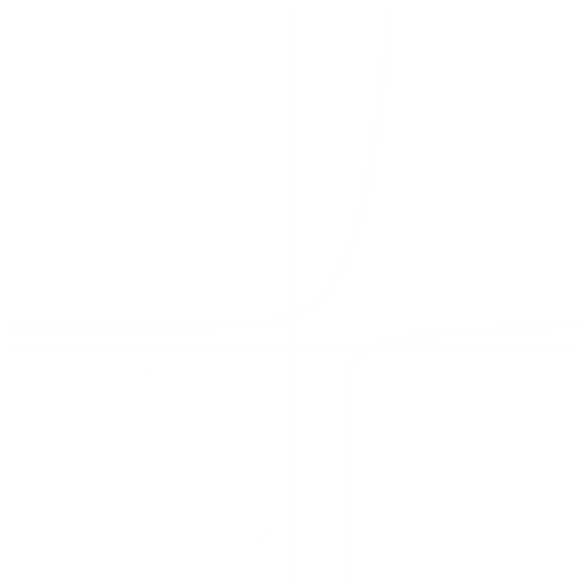
Domain:

Range:

Vertical Asymptote:



Find the inverse of and draw in the same place.



## Trigonometric Functions

Range Domain

Range Domain

Range Domain

Range Domain

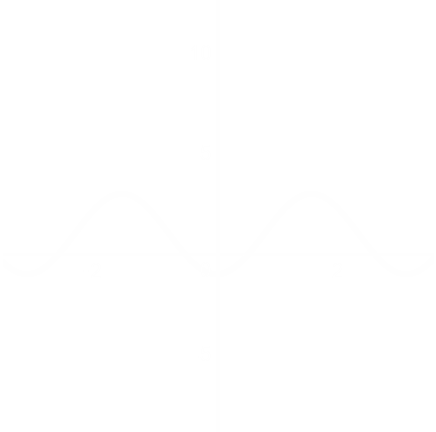
Range Domain

Range Domain

## Graphs of Sinusoidal Functions

Here, is the amplitude and is the period.

Draw the graph of .



Range:

Domain: